RADICAL EXPRESSIONS & EQN'S "CHEAT SHEET"   all math 8-19-08

Radical expressions have square root or other-root groups in them: $\sqrt[4]{(x+4)} + 3$

If it's a square root you undo it by squaring, cube root by cubing, 4^{th} root by taking the 4^{th} power, etc; BUT <u>how hard this is depends directly on how much preparation you do</u>!

Much of the time, you can gather the radical terms into one radical as one side of the equation. This depends upon them being "like radicals" (same argument, same index).

Example - solve: $x = \sqrt[4]{(x+4)} + 3$

If you subtract the 3 from both sides, you have a single radical on one side, and all the "normal" stuff on the other:

$$x - 3 = \sqrt[4]{(x+4)}$$

Now just undo 4^{th} root by taking <u>all of</u> both sides to the 4^{th} power:

$$(x - 3)^4 = (\sqrt[4]{(x+4)})^4$$

4^{th} power undoes 4^{th} root on the right, and the left is just binomial-to-a-power:

$$(x^2-6x+9)^2 = x + 4$$

$$x^4 -6x^3+9x^2 -6x^3+36x^2-54x+9x^2-54x+81 = x+4$$

$$x^4-12x^3+54x^2-108x+81 = x+4$$

Then collect all like terms on one side for a simpler equation to solve:

$$x^4-12x^3+54x^2-109x+77 = 0$$

<u>No obvious factoring solutions</u> suggests graphical solution on a graphing calculator - and there are two approximate real solutions by that method:

approximately, x = 1.4706406, and 4.7183373

Both of these must be checked in the original equation to screen out any "extraneous solutions":

$$x = \sqrt[4]{(x+4)} + 3$$

$$1.4706406 =? \sqrt[4]{(1.4706406+4)} + 3 = 4.529359362 \quad \text{NO!}$$

$$4.7183373 =? \sqrt[4]{(4.7183373+4)} + 3 = 4.718337298 \quad \text{yes}$$

x = 1.4706406 is clearly not a valid solution of the radical equation

x = 4.7183373 clear is a solution, at least within the round-off error of a calculator-approximation solution

NOTE: there is also a complex-conjugate pair of solutions, which for some problems would also have to be checked in the original radical equation. <u>However, most problems are looking for real-number answers</u>.

Sometimes, you cannot gather everything into a single radical on one side of the equation. Gather what radicals you can as terms on one side, with "normal stuff" on the other, then square <u>all of</u> both sides, and collect like terms. Now you can gather the remaining radical term onto one side and square again, <u>if it has a variable in it</u>. From here, gather your terms into a standard polynomial and solve it. (<u>If not</u>, just factor and solve.) Then check your answers in the original radical equation.

Example:

$$\sqrt{x} + \sqrt{2x} = 2$$

$$(\sqrt{x} + \sqrt{2x})^2 = 2^2$$

$$(\sqrt{x})^2 + 2\sqrt{x}\,\sqrt{2x} + (\sqrt{2x})^2 = 4$$

$$x + 2\sqrt{2x^2} + 2x = 4$$

$$3x + 2\sqrt{2}\,x = 4$$

(we can just factor this to solve, no need to square again, <u>because there is no x under the remaining radical</u>)

$$x(3 + 2\sqrt{2}) = 4$$

$$x = \frac{4}{(3 + 2\sqrt{2})} = 0.686291501 \text{ approximately}$$

and plugging this value back into the original radical equation works, to within calculator round-off