RATIONAL FN'S & EQN'S "CHEAT SHEET" **MATH1314/GWJ/08**

Rational = ratio = fraction

Rational number = one real number divided by another, as in 2/3, = constant/constant

Generalize constants to polynomials: p(x)/q(x) = rational function, as in (2x+1)/(3x – 4)

GRAPHING (MATH 1314 only):

There are five things to look for:
(1) common factors top & bottom (hole in graph)
(2) vertical asymptotes where the denominator = 0
(3) x-intercepts where the numerator = 0
(4) y-intercept at x=0
(5) any horizontal (HA) or slant (SA) asymptotes

The best graphing procedure:
(1) factorize the numerator and denominator first
(2) take note of x's where cancelling factors create holes
(3) solve for all VA x's that make denominator = 0 (easy if already factorized)
(4) determine the y-intercept by substituting x = 0
(5) determine x-intercept locations
(6) determine any horizontal (HA) or slant (SA) asymptotes
(7) add just enough table-of-values points to the VA / x-intercept / HA / SA information to graph (test between the critical points: VA's and x-intercepts)
(8) compare your manual picture to the graphing calculator

Example:

given: $(x^3+2x^2-11x-12)/(x^2-x-2)$
lead coeff. ratio $x^3/x^2 = x$ says we have a slant asymptote SA
lead coeff. ratio also says function < 0 at x << 0, function > 0 at x >> 0
factorize: {(x-3)(x+1)(x+4)}/{(x+1)(x-2)}
cancel (x+1)'s (hole at x = -1): {(x-3)(x+4)}/(x-2)
determine y-intercept at x = 0: -12/(-2) = +6
determine x-intercepts from numerator factors: x = 3, x = -4
re-multiply simplified numerator: $(x^2+x-12)/(x-2)$
zero in denominator at x = 2, that is the VA
do the division (see below): = (x+3), r = -6
pitch the remainder: SA is y = x+3
can always test more points, but that is not needed in this example

this graph has VA at x = 2, x-intercepts at -4 and +3, y-intercept at 6, a hole at x = -1, and approaches SA y = x + 3 from above on the left, and from below on the right, starting from x = 0

COMBINING RATIONAL EXPRESSIONS (all math classes):

Multiplication
(1) factorize everything
(2) simplify by cancelling if possible
(3) "multiply" factors across top and bottom

Addition/subtraction
(1) factorize everything
(2) determine a common denominator in factored form
(3) from CD, determine "Judo-1 factors" for each term
(4) add the numerator terms up over the CD
(5) recombine numerator polynomial
(6) re-factorize numerator (if possible)
(7) re-simplify by further cancelling (if possible)

Division:
(1) factorize everything
(2) invert the divisor rational expression and write it as a multiply
(3) run the multiply procedure

SIMPLIFYING COMPLEX FRACTIONS (all math classes):

If the individual numerator and denominator expressions are fairly simple

1. Use the addition/subtraction procedure to convert the numerator and the denominator into "simple fraction" rational expressions
2. Run the division procedure

If the individual numerator and denominator expressions are fairly complex

1. Determine an overall common denominator for every term in both the numerator and denominator.
2. Use that overall common denominator as a "Judo-1" factor for the entire complex fraction
3. Multiply-out the numerator and the denominator, which converts complex fraction to an ordinary rational expression

SOLVING RATIONAL EQUATIONS (all math classes):

(1) factorize everything
(2) determine domain exclusions ("forbidden-x list") from denominator factors with variables in them
(3) determine LCD (or any CD)
(4) multiply thru by CD: this creates an ordinary polynomial to solve
(5) solve what remains by normal polynomial methods
(6) discard any solutions that are on the forbidden-x list (if you have to discard them all, then the correct answer is "no solution")
(7) check your surviving answers in the original rational equation

Example: given: $3/(x+4) + 2/x = 7$ (same as 7/1 !!!)

Domain exclusions (FXL): $x = -4$ and $x = 0$

GCD: (x+4)'s times x's times 1's = (x+4)x

Mult thru by GCD: $\frac{3(x+4)x}{(x+4)} + \frac{2(x+4)x}{x} = \frac{7(x+4)x}{1}$

Simplify (cancel): $3x + 2(x+4) = 7(x+4)x$

Distribute & collect like: $3x + 2x + 8 = 7x^2 + 28x$

$5x + 8 = 7x^2 + 28x$

$7x^2 + 23x - 8 = 0$

Solve by quad formula: $D = 23^2 – 4(7)(-8) = 529 + 224 = 753$

$x = -23/(2*7) +/- \sqrt{753} / (2*7)$

$x = -23/14 +/- \sqrt{753} /14$

approximately, $x = -3.603, +0.317$

Note that neither solution occurs at the "forbidden" zero-denominator points of x = 0 and x = -4; & that graphing sol'n gets same answers!

POLYNOMIAL LONG DIVISION (all math classes):

<table>
<tr><th>With numbers, as
in elementary school</th><th>Description</th><th>With polynomials</th></tr>
<tr><td>27 | 91</td><td>setup</td><td>(x+1) | x^2-x-3</td></tr>
<tr><td>3
27 | 91</td><td>Guess how many times
divisor (lead term) will
go into dividend (lead term)</td><td>x
(x+1) | x^2-x-3</td></tr>
<tr><td>3
27 | 916
81</td><td>Multiply quotient times
divisor and write it under
dividend (line it up!)</td><td>x
(x+1) | x^2-x-3
x^2+x</td></tr>
<tr><td>3
27 | 916
-(81)
10</td><td>Subtract as shown</td><td>x
(x+1) | x^2-x-3
-(x^2+x)
-2x</td></tr>
<tr><td>3
27 | 916
-(81)
106</td><td>Bring down the next
digit (term)</td><td>x
(x+1) | x^2-x-3
-(x^2+x)
-2x-3</td></tr>
<tr><td>34
27 | 916
-(81)
106</td><td>Guess the next digit (term)
in the quotient</td><td>x-2
(x+1) | x^2-x-3
-(x^2+x)
-2x-3</td></tr>
<tr><td>33
27 | 916
-(81)
106
-(81)
25</td><td>Multiply and subtract as
before</td><td>x-2
(x+1) | x^2-x-3
-(x^2+x)
-2x-3
-(-2x-2)
-1</td></tr>
<tr><td>916/27 = 33, r = 25</td><td>Express in remainder form</td><td>$\frac{(x^2-x-3)}{(x+1)} = (x-2), r = -1$</td></tr>
<tr><td>916/27 = 33 + 25/33</td><td>Or in fraction form</td><td>$\frac{(x^2-x-3)}{(x+1)} = (x-2) - \frac{1}{(x+1)}$</td></tr>
</table>

SYNTHETIC DIVISION (required for MATH 1314 ONLY):

For synthetic division, which ONLY works for linear factors as divisors, you reverse the sign on the constant in the linear factor divisor, with a multiply-and-add procedure:

Same problem (x+1) | $\overline{x^2\text{-}x\text{-}3}$ becomes

```
-1 | 1 -1 -3
```

Bring down the first number below what will be an addition line as an "addition result"

```
-1 | 1 -1 -3
     _______
     1
```

Multiply divisor (-1) by the addition result (1) and place that product above the addition line, next place

```
-1 | 1 -1 -3
     __-1__
     1
```

Do the addition

```
-1 | 1 -1 -3
     __-1___
     1 -2
```

Multiply divisor times 2nd result, place result above addition line, and do that indicated addition

```
-1 | 1 -1 -3
     __-1+2_
     1 -2 -1
```

The last digit (-1) is the remainder, while the 1 and the 2 are the coefficients in the quotient polynomial, in this case 1x-2, or just x-2

And again the answer may be expressed in remainder form (x-2), r = -1

or it may be in fraction form (x-2) – 1/(x+1), whichever you need